Model Answer   Mid-Term Exam   Math. 1   November 2015   15 Marks

[1](a) $y = 2x^3 + 3\cos 3x$

$$y` = 6x^2 - 9\sin 3x$$

(b) $y = \sqrt{x} + 2\tan x^2$

$$y` = \frac{1}{2}x^{-\frac{1}{2}} + 2\sec^2 x^2 \,.\, 2x$$

(c) $y = (x + \sec x)^4$

$$y` = 4(x + \sec x)^3.(1 + \sec x \,.\tan x)$$

(d) $\mathrm{y} = \sin \mathrm{x}^4 + \sin^4 \mathrm{x}$

$$\mathrm{y}` = \cos \mathrm{x}^4 \,.\, 4\mathrm{x}^3 + 4\sin^3\mathrm{x}.\cos \mathrm{x}$$

(e) $y = \cos\frac{2}{x}.\tan\frac{x}{2}$

$$y` = \cos\frac{2}{x}.\sec^2\frac{x}{2}.\frac{1}{2} - \sin\frac{2}{x}.\left(-\frac{2}{x^2}\right).\tan\frac{x}{2}$$

(f) $y = \frac{3+\cos x}{x+\tan x}$

$$y` = \frac{(x+\tan x).(0-\sin x)-(3+\cos x)(1+sex^2 x)}{(x+\tan x)^2}$$

--   6-Marks

[2](a) $\lim\limits_{\mathrm{x}\to 1}\frac{\sqrt[3]{\mathrm{x}}-1}{\mathrm{x}^5-1} = \frac{0}{0} = \frac{1/3}{5} = \frac{1}{15}$

(b) $\lim\limits_{\mathrm{x}\to 0}\frac{\mathrm{x}^5+\sin^4\mathrm{x}}{\mathrm{x}^4+\tan^4\mathrm{x}} = \frac{0}{0} = \lim\limits_{\mathrm{x}\to 0}\frac{\mathrm{x}+\frac{\sin^4\mathrm{x}}{\mathrm{x}^4}}{1+\frac{\tan^4\mathrm{x}}{\mathrm{x}^4}} = \frac{0+1}{1+1} = \frac{1}{2}$

(c) $\lim\limits_{\mathrm{x}\to 0}\frac{\mathrm{x}-\sin\mathrm{x}}{\mathrm{x}-\tan\mathrm{x}} = \frac{0}{0} = \lim\limits_{\mathrm{x}\to 0}\frac{1-\cos\mathrm{x}}{1-\sec^2\mathrm{x}} = \frac{0}{0}$

$$= \lim_{\mathrm{x}\to 0}\frac{\sin\mathrm{x}}{-2\sec^2\mathrm{x}.\tan\mathrm{x}} = \lim_{\mathrm{x}\to 0}\frac{\cos^3\mathrm{x}}{-2} = -\frac{1}{2}$$

(d) $\lim\limits_{x\to\infty}\sqrt{\dfrac{x^3-2x}{3+4x^3}}=\dfrac{\infty}{\infty}=\lim\limits_{x\to\infty}\sqrt{\dfrac{1-\dfrac{2}{x^2}}{\dfrac{3}{x^3}+4}}=\sqrt{\dfrac{1}{4}}=\dfrac{1}{2}$

-- 4-Marks

[3](a) $f(x)=x^3-3x^2-9x+1,\ f`(x)=3x^2-6x-9=0.$

Then (x – 3)(x + 1) = 0.

Then x = 3 , x = – 1 are critical points. Also, $f``(x)=6x-6=0.$

Since $f``(3)=18-6=12$, then 3 is minimum.

$f``(-1)=-6-6=-12$, then – 1 is maximum.

-- 3-Marks

(b)Since $\cos x=1-\frac{1}{2!}x^2+\frac{1}{4!}x^4-\cdots.$

Then $\cos 3x=1-\frac{1}{2!}(3x)^2+\frac{1}{4!}(3x)^4-\cdots.$

Then $f(x)=x.\cos 3x=x(1-\frac{1}{2!}(3x)^2+\frac{1}{4!}(3x)^4-\cdots)$

$=x-\frac{9}{2!}x^3+\frac{81}{4!}x^5-\cdots.$

-- 2-Marks

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